Probability and Random Processes

ECS 315

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8 Discrete Random Variable

Office Hours:

BKD, 4th floor of Sirindhralai building

Monday	9:30-10:30
Monday	14:00-16:00
Thursday	16:00-17:00

Discrete Random Variable

- X is a **discrete** random variable if it has a countable support.
  - Recall that countable sets include finites set and countably infinite sets.
- For X whose support is uncountable, there are two types:
  - **Continuous** random variable
  - **Mixed** random variable

Probabilities involving discrete RV

- Back to example of rolling a dice
- The "important" probabilities are

$$P[X=1]=P[X=2]=\cdots=P[X=6]=\frac{1}{6}$$

- In tabular form:

Dummy variable

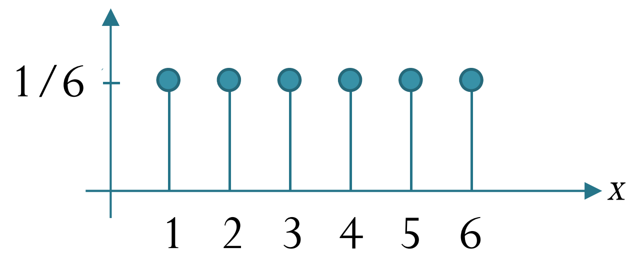
x	$P[X=x]$
1	1/6
2	1/6
3	1/6
4	1/6
5	1/6
6	1/6

- **Probability mass function (PMF)**:

$$p_X(x)=\begin{cases}1/6, & x=1,2,3,4,5,6,\\ 0, & \text{otherwise.}\end{cases}$$

  - In general, $p_X(x)\equiv P[X=x]$

- Stem plot:



Probabilities involving discrete RV

To find P[some condition(s) on X] from the pmf $p_X(x)$ of X:

1. Find the support of X.
2. Consider only the x inside the support.
   Find all values of x that satisfies the condition(s).
3. Evaluate the pmf at x found in the previous step.
4. Add the pmf values from the previous step.

Back to the dice roll example. Suppose we want to find $P[X > 4]$.

1. The support of X is $\{1,2,3,4,5,6\}$.
2. The members which satisfies the condition ">4" is 5 and 6.
3. The pmf values at 5 and 6 are all 1/6.
4. Adding the pmf values gives $2/6 = 1/3$.

Review

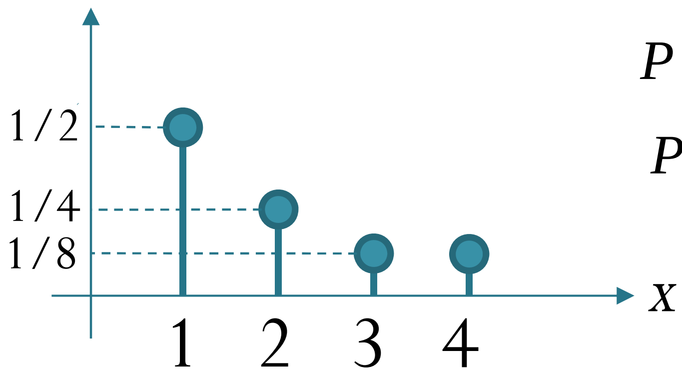
Consider a random variable (RV) X.

probability mass function (pmf)

$$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3, 4\} \\ 0, & \text{otherwise} \end{cases}$$

$$p_X(x) = \mathrm{P}[X = x]$$

stem plot:



$$P[X = 2] = ?$$

$$P[X > 1] = ?$$

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Consider a random variable (RV) X.

probability mass function (pmf) $$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3, 4\} \\ 0, & \text{otherwise} \end{cases}$$

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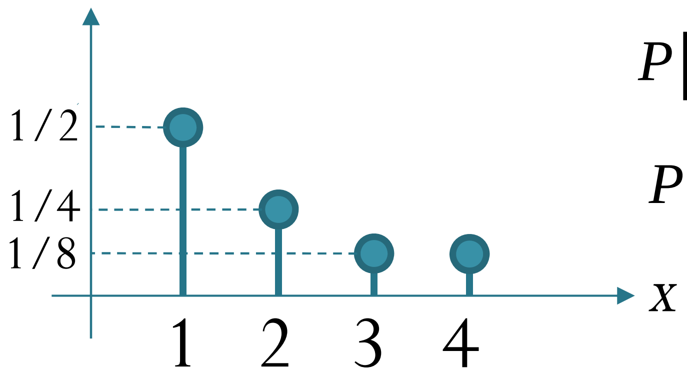

$$P[X = 2] = p_X(2) = \frac{1}{4}$$

$$P[X > 1] = p_X(2) + p_X(3) + p_X(4)$$
$$= \frac{1}{4} + \frac{1}{8} + \frac{1}{8} = \frac{1}{2}$$

Review

Consider a random variable (RV) X.

probability mass function (pmf) ?

$$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

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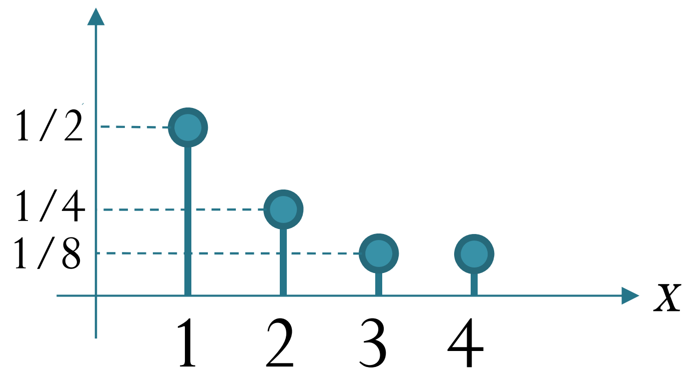

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probability mass function (pmf)

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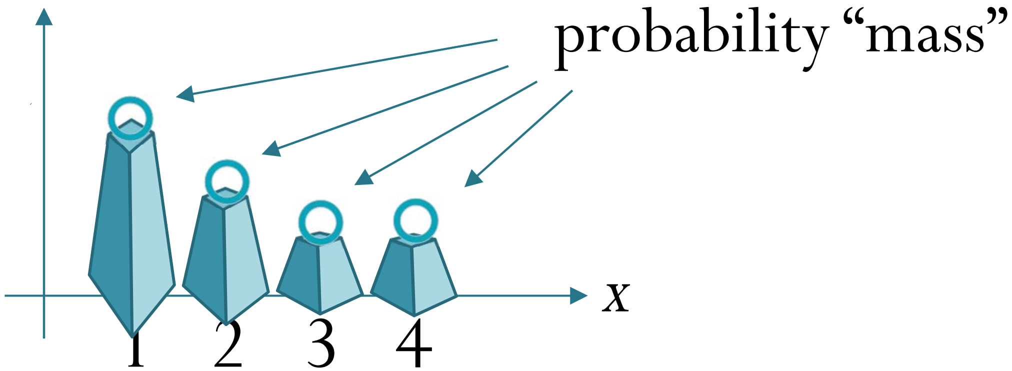

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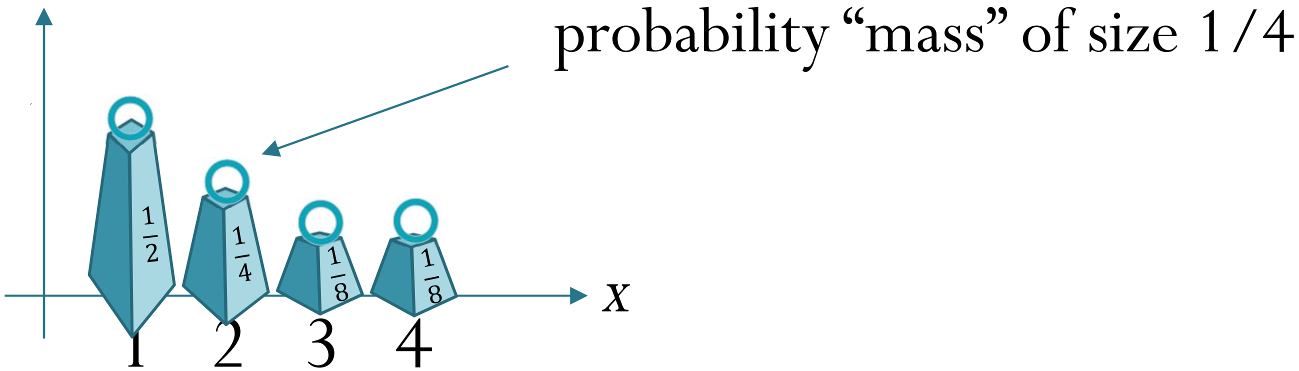

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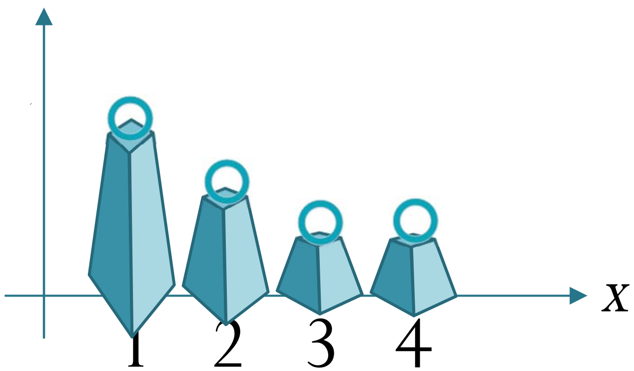


What about the support of this RV X?

Review

Consider a random variable (RV) X.

probability mass function (pmf)

$$p_X(x) = \begin{cases} \frac{1}{2}, & x = 1, \\ \frac{1}{4}, & x = 2, \\ \frac{1}{8}, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

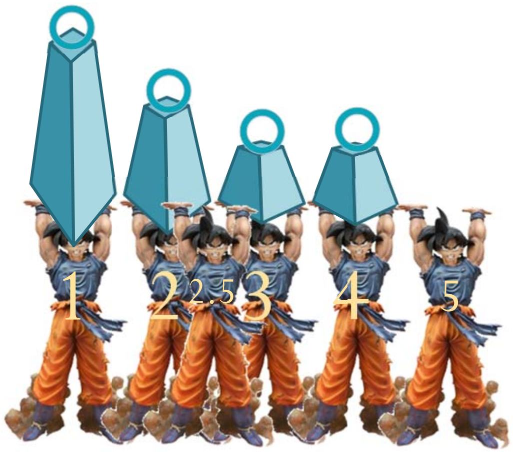
The set $\{1,2,3,4\}$ is a support of X.

Review

Consider a random variable (RV) X.

probability mass function (pmf)

$$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$



The set $\{1, 2, 2.5, 3, 4, 5\}$ is also a support of this RV X.

Review

Consider a random variable (RV) X.

probability mass function (pmf)

$$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3,4\} \\ 0, & \text{otherwise} \end{cases}$$

The set $\{1,2,4\}$ is *not* a support of this RV X.

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The set $\{1,2,3,4\}$ is the "minimal" support of X.

For discrete RV, we take the collection of x values at which $p_X(x) > 0$ to be our **"default" support**.

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Consider a random variable (RV) X.

probability mass function (pmf)

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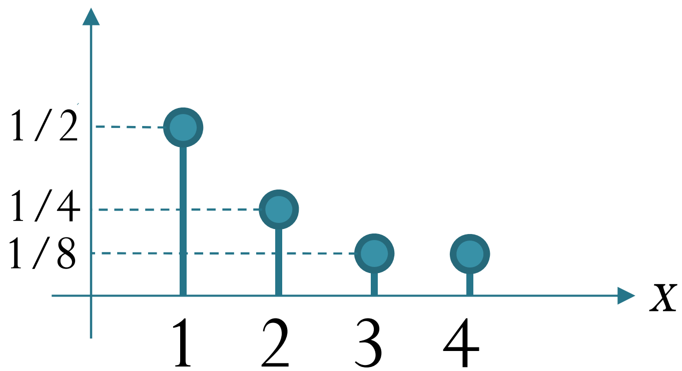
$$p_X(x) = P[X = x]$$

stem plot:



The "default" support for this RV is the set $S_X = \{1,2,3,4\}$.

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$$p_X(x) = \begin{cases} 1/2, & x = 1, \\ 1/4, & x = 2, \\ 1/8, & x \in \{3, 4\} \\ 0, & \text{otherwise} \end{cases}$$

$$p_X(x) = \mathrm{P}[X = x]$$

cumulative distribution function (cdf)

$$F_X(x) = \mathrm{P}[X \le x]$$

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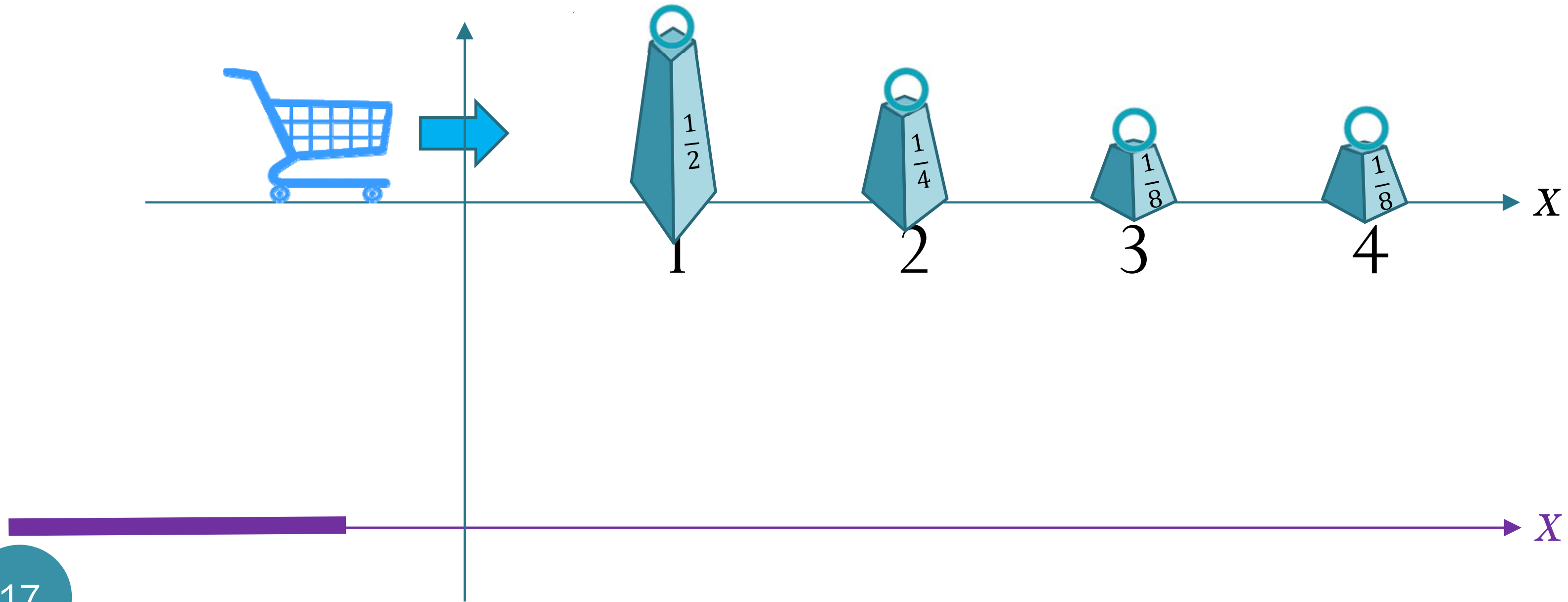

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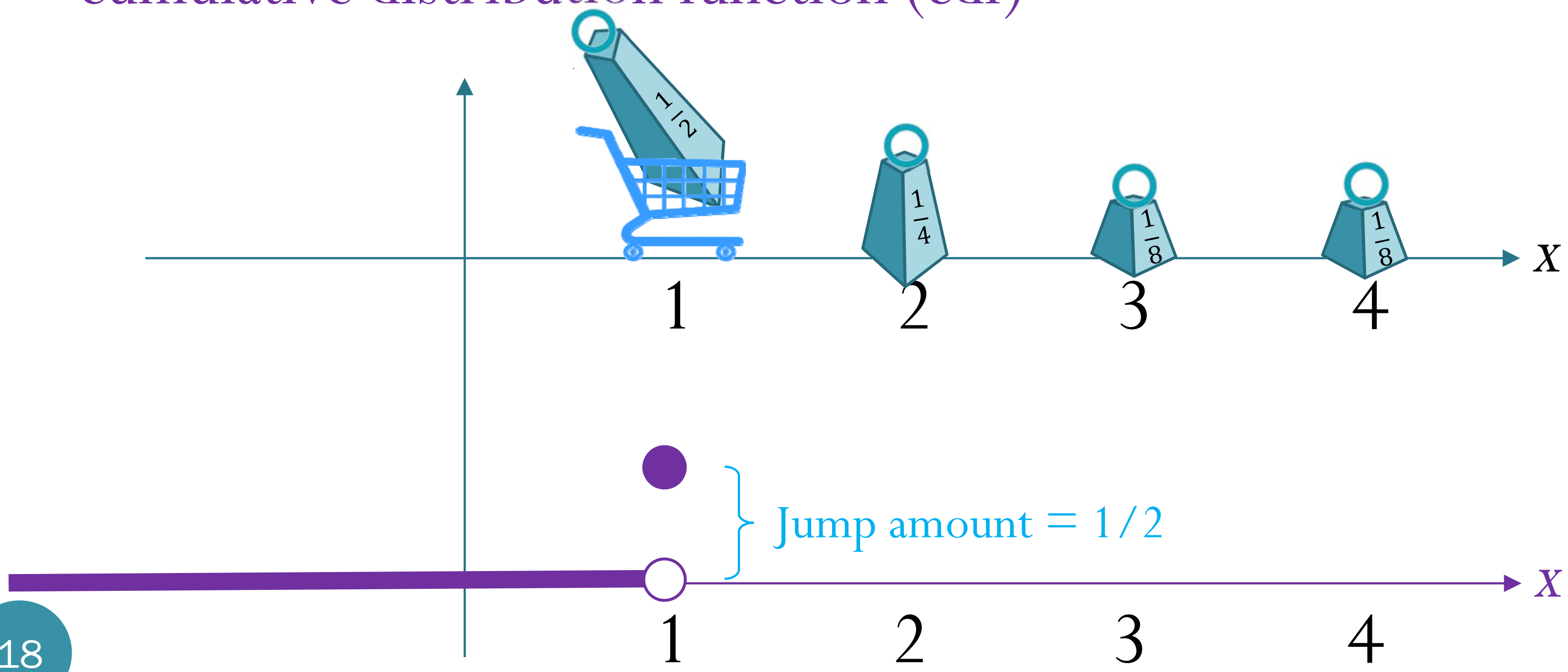

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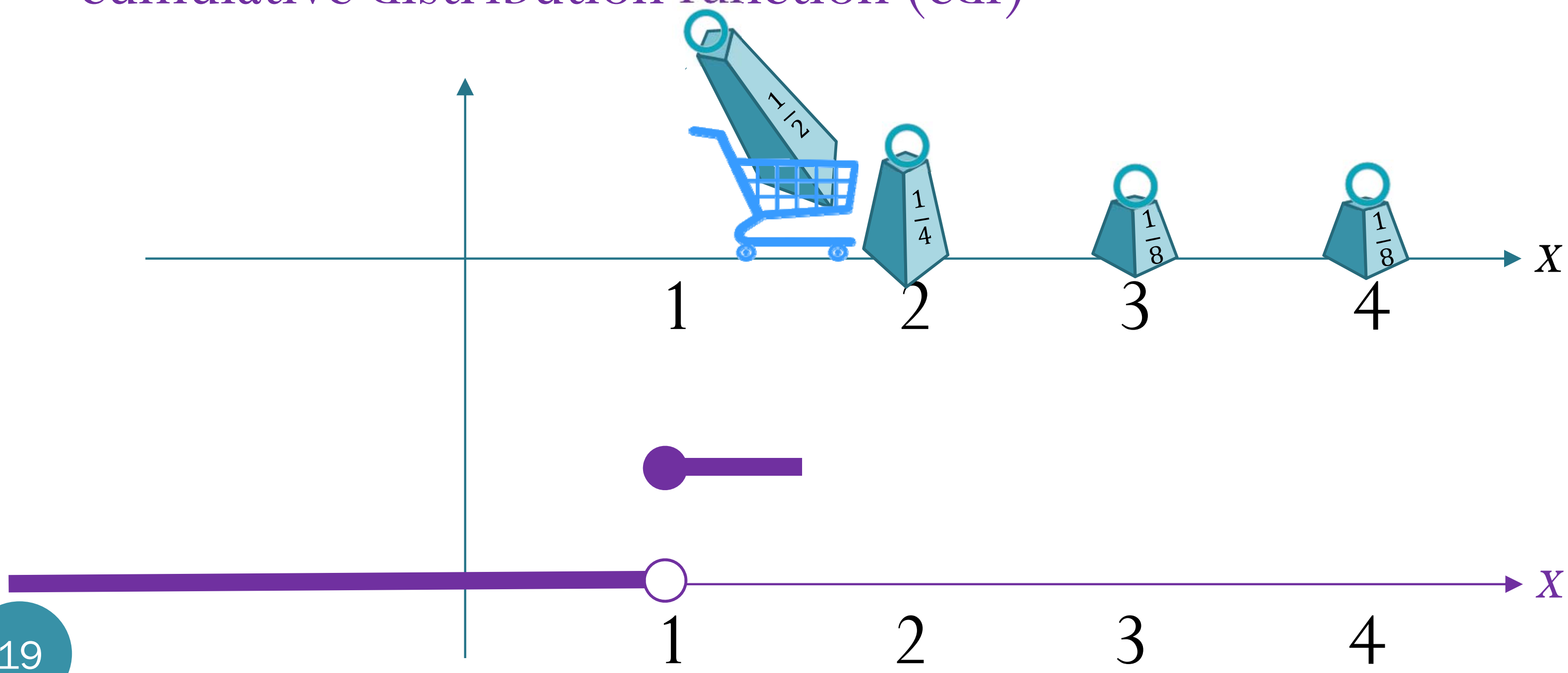

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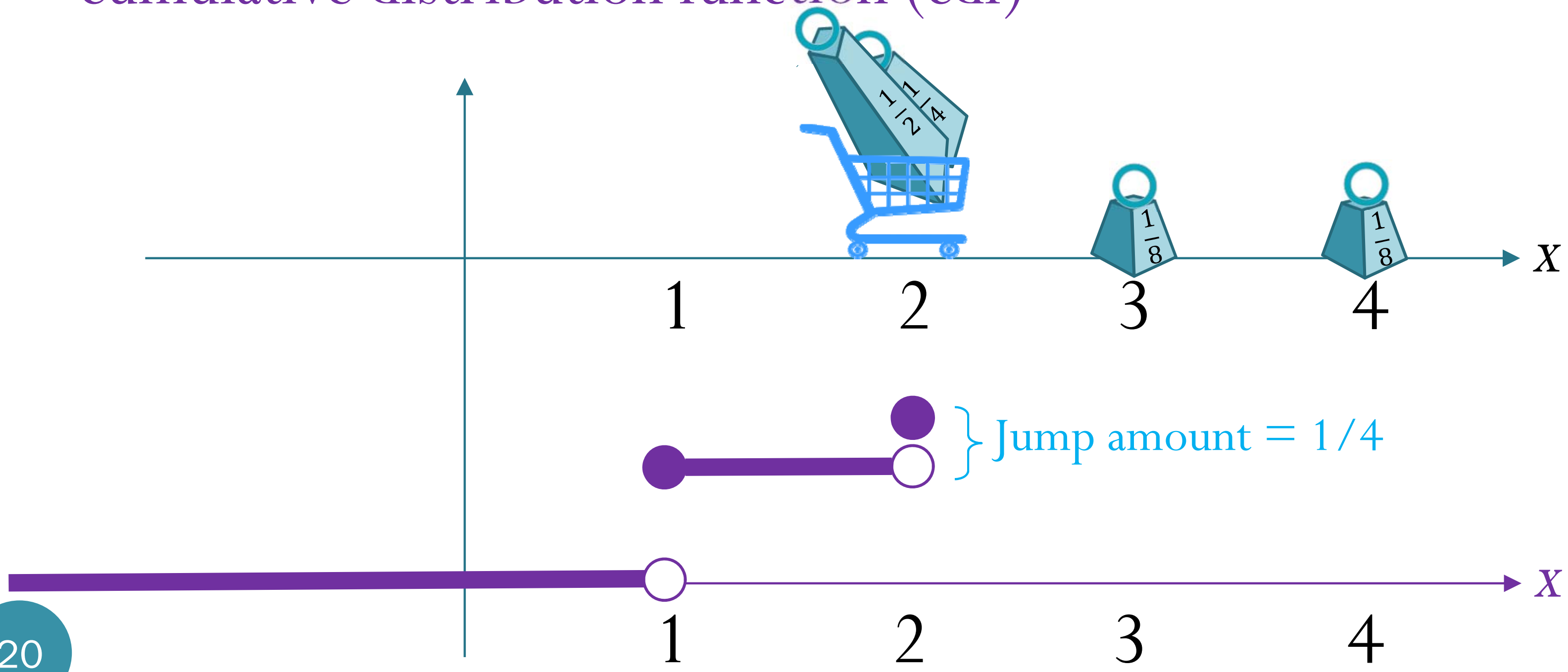

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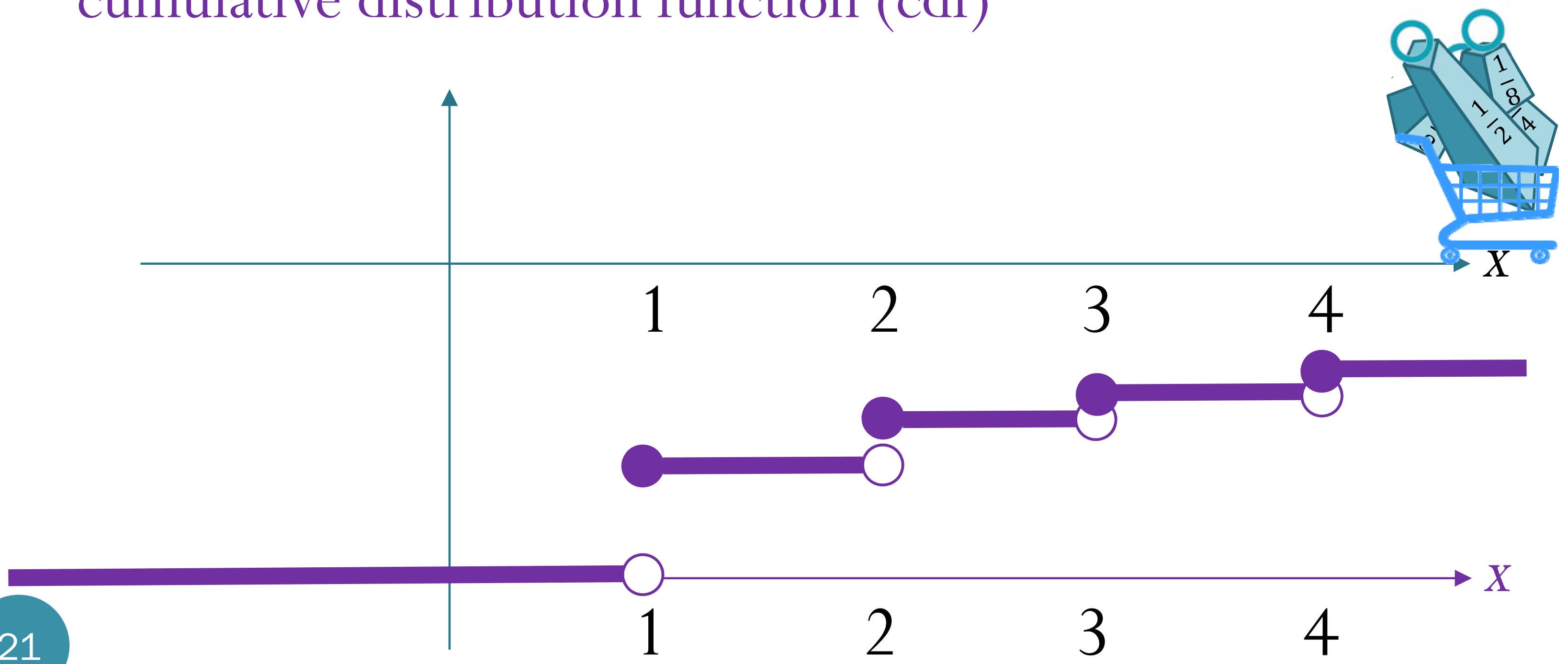

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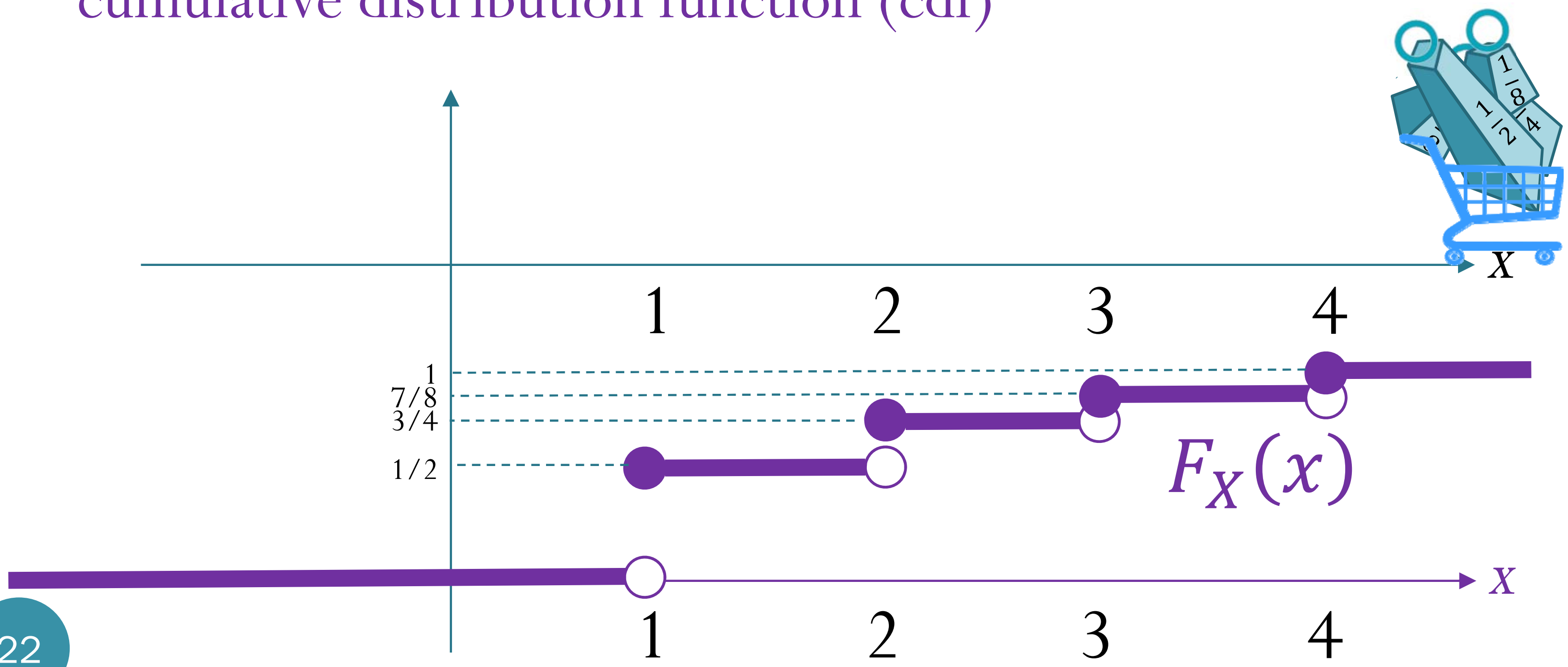

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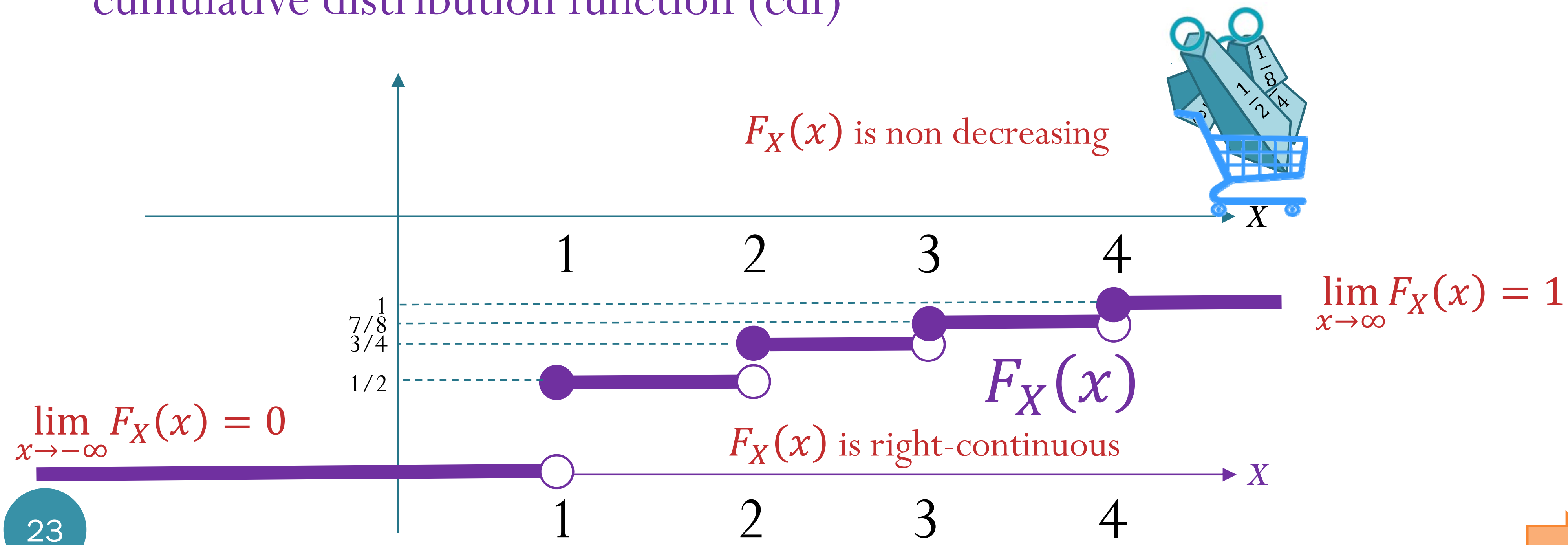

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cumulative distribution function (cdf)

$$F_X(x) = \begin{cases} 0, & x < 1, \\ 1/2, & 1 \le x < 2, \\ 3/4, & 2 \le x < 3, \\ 7/8, & 3 \le x < 4, \\ 1, & x \ge 4. \end{cases}$$